Solutions

MAT 1320 A **Assignment 3** (Due Wed. Nov. 3rd, 8:30) **Student Number:**

Problem 1: §3.5 #8

Work: $y^5 + x^2y^3 = 1 + ye^{x^2}$

then $\frac{d}{dx}(y^5 + x^2y^3) = \frac{d}{dx}(1 + ye^{x^2})$, so $5y^4\frac{dy}{dx} + 2xy^3 + 3x^2y^2\frac{dy}{dx} = e^{x^2}\frac{dy}{dx} + 2xye^{x^2}$

Answer: $\frac{dy}{dx} =$ $\boxed{\frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}}$

Problem 2: §3.6 #22

Work: $f(x) = x\ln(\arctan x)$

$f'(x) = \ln(\arctan x) + x\left(\frac{1}{\arctan x}\right)\left(\frac{1}{1+x^2}\right)$

Answer: $f'(x) =$ $\boxed{\ln(\arctan x) + \frac{x}{(1+x^2)\arctan x}}$

Problem 3: §3.7 #12

Work: $h(x) = \ln\left(x + \sqrt{x^2-1}\right)$

So $h'(x) = \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{1}{2}(x^2-1)^{-1/2}(2x)\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\left(1 + \frac{x}{\sqrt{x^2-1}}\right)$

$= \frac{1}{x+\sqrt{x^2-1}}\left(\frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}\right)$

Answer: $h'(x) =$ $\boxed{\frac{1}{\sqrt{x^2-1}}}$

Problem 4: §3.7 #36

Work: $y = \sqrt[4]{\dfrac{x^2+1}{x^2-1}} = \dfrac{(x^2+1)^{1/4}}{(x^2-1)^{1/4}}$

so $\ln(y) = \ln\left(\dfrac{(x^2+1)^{1/4}}{(x^2-1)^{1/4}}\right) = \ln\left((x^2+1)^{1/4}\right) - \ln\left((x^2-1)^{1/4}\right)$

$= \frac{1}{4}\left(\ln(x^2+1) - \ln(x^2-1)\right)$

then $\dfrac{d}{dx}(\ln y) = \dfrac{d}{dx}\left(\frac{1}{4}\left(\ln(x^2+1) - \ln(x^2-1)\right)\right)$

giving $\dfrac{1}{y}\dfrac{dy}{dx} = \frac{1}{4}\left(\dfrac{2x}{x^2+1} - \dfrac{2x}{x^2-1}\right) = \dfrac{x}{2}\left(\dfrac{x^2-1-(x^2+1)}{(x^2+1)(x^2-1)}\right) = \dfrac{-x}{x^4-1}$

then $\dfrac{dy}{dx} = \left(\dfrac{x}{1-x^4}\right)(y)$

Answer: $\dfrac{dy}{dx} = \boxed{\left(\dfrac{x}{1-x^4}\right)\sqrt[4]{\dfrac{x^2+1}{x^2-1}}}$

Problem 5: §3.9 #10

Work: $g(x) = \sqrt[3]{1+x} = (1+x)^{1/3}$ at $a = 0$

$g'(x) = \frac{1}{3}(1+x)^{-2/3}$, so $g'(0) = 1/3$ and $g(0) = 1$

tangent line is $y - 1 = 1/3\,(x-0) \Rightarrow y = \frac{1}{3}x + 1$

so $\sqrt[3]{0.95} = \sqrt[3]{1+(-0.05)} \simeq \frac{1}{3}(-0.05) + 1$

and $\sqrt[3]{1.1} = \sqrt[3]{1+(0.1)} \simeq \frac{1}{3}(0.1) + 1$

Answers: $L(x) = \boxed{\frac{1}{3}x + 1}$ $\quad \sqrt[3]{0.95} \approx \boxed{0.9833}$ $\quad \sqrt[3]{1.1} \approx \boxed{1.0333}$

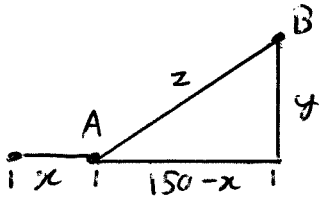
Problem 6: §4.1 # 12

Work:

at noon

A ——— 150km ——— B

later, if ship A has travelled x km east and ship B y km north



we're told that $\frac{dx}{dt} = 35$ km/h

and $\frac{dy}{dt} = 25$ km/h

Let z be the distance between them,

then we want $\frac{dz}{dt}$ when $x = 140$ km and $y = 100$ km

(at $t = 4$ hrs)

by Pythagoras $z^2 = y^2 + (150 - x)^2$

so $2z\frac{dz}{dt} = 2y\frac{dy}{dt} - 2(150 - x)\frac{dx}{dt}$

thus $\frac{dz}{dt} = \frac{y\frac{dy}{dt} - (150 - x)\frac{dx}{dt}}{z} = \frac{(100)(25) - (150-140)(35)}{\sqrt{(10)^2 + (100)^2}}$

Answer: 21.39 km/h

Problem 7: §4.2 # 52

Work: $f(x) = x - 2\arctan x$ on $[0, 4]$

$f'(x) = 1 - \frac{2}{1+x^2} = 0$ if $x = \pm 1$ (-1 not in interval)

then $f(0) = 0$

$f(1) = 1 - 2\arctan(1) = 1 - 2(\pi/4) = 1 - \pi/2 \simeq -0.5708$

$f(4) = 4 - 2\arctan(4) \simeq 1.3484$

Answers: abs. max is $4 - 2\arctan(4)$ or 1.3484 abs. min is $1 - \pi/2$ or -0.5708