$\boxed{\text{Solutions}}$

MAT 1320 A **Assignment 6** (Due Wed. Dec. 8th, 10:00) **Student Number:**

Problem 1: §5.3 #14

Work:

$$\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta = \sec\theta \Big|_0^{\pi/4} = \sec(\pi/4) - \sec(0)$$

Answer: $\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta =$ $\boxed{\sqrt{2} - 1} \simeq \boxed{0.4142}$

Problem 2: §5.3 #22

Work:

$$\int_0^1 \frac{4}{t^2+1} \, dt = 4\arctan t \Big|_0^1$$
$$= 4(\arctan(1) - \arctan(0))$$
$$= 4(\pi/4 - 0)$$

Answer: $\int_0^1 \frac{4}{t^2+1} \, dt =$ $\boxed{\pi} \simeq \boxed{3.1416}$

Problem 3: §5.4 #16

Work:

$$y = \int_{e^x}^0 \sin^3 t \, dt = -\int_0^{e^x} \sin^3 t \, dt$$
$$\frac{dy}{dx} = -\frac{d}{dx}\left(\int_0^{e^x} \sin^3 t \, dt\right) = -\sin^3(e^x)\left(\frac{d}{dx}(e^x)\right)$$

Answer: $\frac{d}{dx}\left(\int_{e^x}^0 \sin^3(t) \, dt\right) =$ $\boxed{-e^x \sin^3(e^x)}$

Problem 4: §5.5 #34

Work: $\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2}$

$\left(\text{let } \begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array} \right) = -\arctan(u) + C$

Answer: $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx =$ $\boxed{-\arctan(\cos x) + C}$

Problem 5: §5.5 # 48

Work: $\int_0^{\pi/2} \cos x \,\sin(\sin x)\,dx = \int_0^1 \sin(u)\,du$

$\left(\begin{array}{l} \text{let } u = \sin x \\ du = \cos x\,dx \\ x = 0 \to u = 0 \\ x = \pi/2 \to u = 1 \end{array} \right)$ $= -\cos(u)\Big|_0^1$

$= -(\cos(1) - \cos(0))$

Answer: $\displaystyle\int_0^{\pi/2} \cos x \sin(\sin x)\,dx =$ $\boxed{1 - \cos(1)}$ $\simeq \boxed{0.4597}$

Problem 6: §5.6 # 12

Work: $\int \arcsin x\,dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$ $\left(\begin{array}{l} y = 1-x^2 \\ dy = -2x\,dx \end{array} \right)$

$\left(\begin{array}{ll} u = \arcsin x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}\,dx & v = x \end{array} \right)$ $= x\arcsin x + \frac{1}{2}\int \frac{1}{\sqrt{y}}\,dy$

$= x\arcsin x + \sqrt{y} + C$

Answer: $\displaystyle\int \arcsin x\,dx =$ $\boxed{x\arcsin x + \sqrt{1-x^2} + C}$

Problem 7: §5.6 #18

Work: $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy = 2\sqrt{y}\ln y\Big|_4^9 - \int_4^9 2\sqrt{y}\,\frac{1}{y}\,dy$

$\left(\begin{array}{ll} u = \ln y & dv = \frac{dy}{\sqrt{y}} \\ du = \frac{1}{y}\,dy & v = 2\sqrt{y} \end{array} \right)$ $= 2\sqrt{y}\ln y\Big|_4^9 - \int_4^9 \frac{2}{\sqrt{y}}\,dy$

$= 2\sqrt{y}\ln y\Big|_4^9 - 4\sqrt{y}\Big|_4^9 = 2(3)\ln 9 - 2(2)\ln 4$

$- (4(3) - 4(2))$

Answer: $\displaystyle\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy =$ $\boxed{6\ln 9 - 4\ln 4 - 4}$ $\simeq \boxed{3.6382}$