Solutions

MAT 1320 A **Assignment 4** (Due Wed. Nov. 10th, 8:30) **Student Number**:

Problem 1: §4.3 #34

Answers:

(a) vertical asymptote(s): $\boxed{x=2}$ horizontal asymptote(s): $\boxed{y=1}$

(b) $f'(x) = \boxed{\dfrac{-4x}{(x-2)^3}}$

$f(x)$ increasing on $\boxed{(0,2)}$ $f(x)$ decreasing on $\boxed{(-\infty,0) \text{ and } (2,\infty)}$

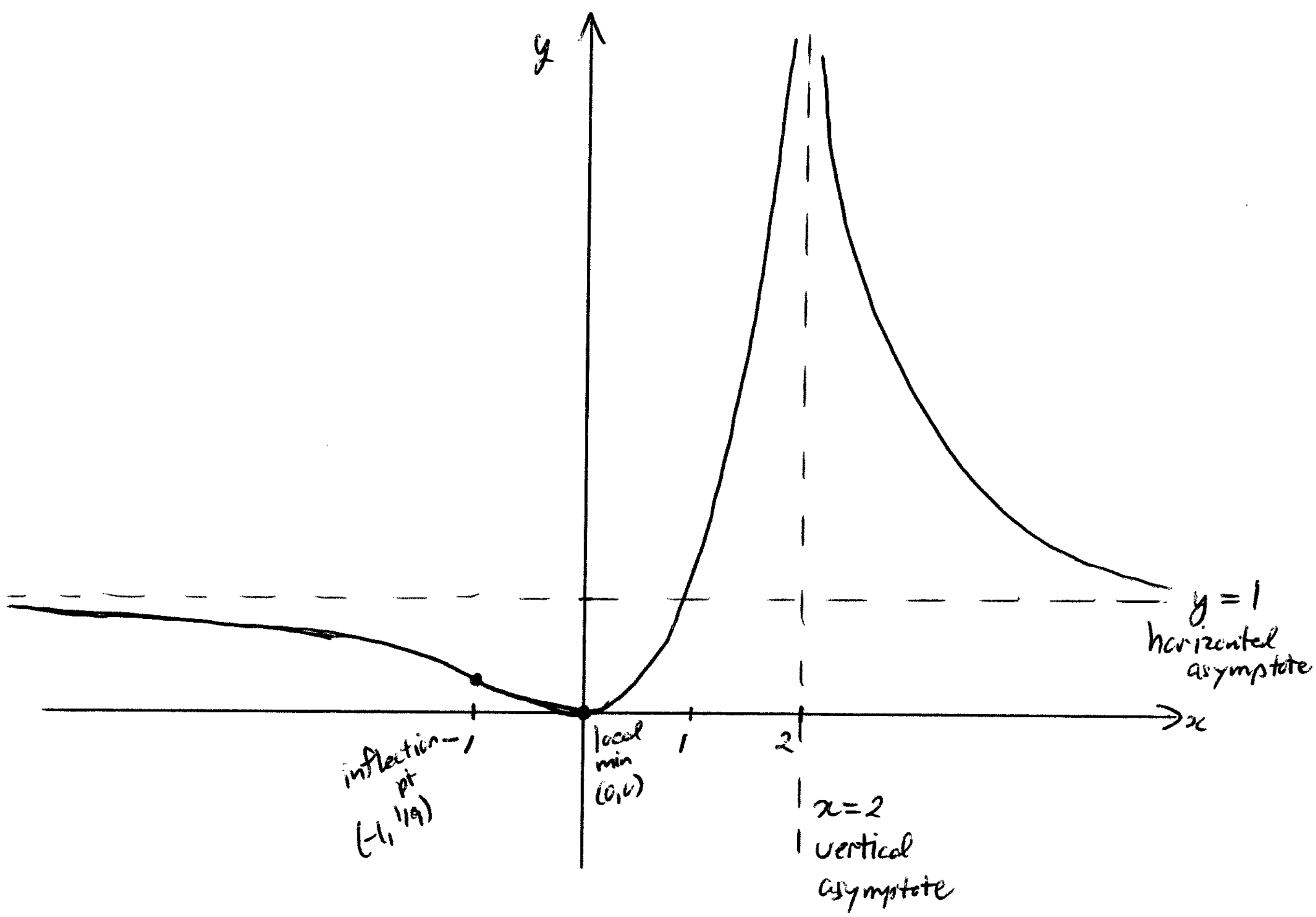
(c) local extrema (points): $\boxed{(0,0)}$ (local min)

(d) $f''(x) = \boxed{\dfrac{8(x+1)}{(x-2)^4}}$

$f(x)$ concave up on $\boxed{(-1,2) \text{ and } (2,\infty)}$ $f(x)$ concave down on $\boxed{(-\infty,-1)}$

inflection points: $\boxed{(-1, 1/9)}$

(e) (sketch)



Problem 2: §4.3 #38

Answers:

(a) vertical asymptote(s): $\boxed{\text{none}}$ horizontal asymptote(s): $\boxed{y=0 \quad \text{and} \quad y=1}$

(b) $f'(x) = \boxed{\dfrac{e^x}{(1+e^x)^2}}$

$f(x)$ increasing on $\boxed{(-\infty, \infty)}$ $f(x)$ decreasing on $\boxed{\text{never}}$

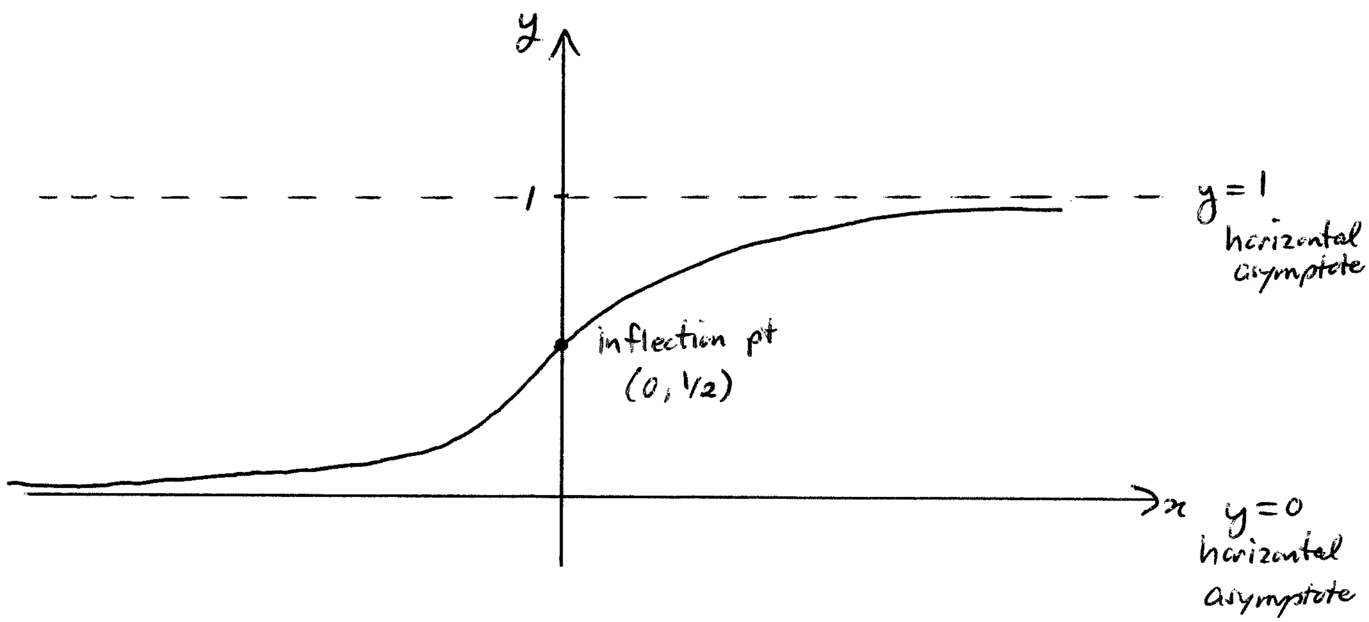
(c) local extrema (points): $\boxed{\text{none}}$

(d) $f''(x) = \boxed{\dfrac{e^x(1-e^x)}{(1+e^x)^3}}$

$f(x)$ concave up on $\boxed{(-\infty, 0)}$ $f(x)$ concave down on $\boxed{(0, \infty)}$

inflection points: $\boxed{(0, 1/2)}$

(e) (sketch)



Problem 3: §4.5 #20

Work:

$$\lim_{x\to 0} \frac{\cos(mx)-\cos(nx)}{x^2} = \lim_{x\to 0} \frac{-m\sin(mx)+n\sin(nx)}{2x} = \lim_{x\to 0} \frac{-m^2\cos(mx)+n^2\cos(nx)}{2}$$

$(0/0)$ (still $0/0$)

Answer: $\boxed{\frac{1}{2}(n^2-m^2)}$

Problem 4: §4.5 #32

Work:

$$\lim_{x\to\infty} x\tan(1/x) = \lim_{x\to\infty} \frac{\tan(1/x)}{1/x} = \lim_{x\to\infty} \frac{\sec^2(1/x)(-1/x^2)}{(-1/x^2)} = \lim_{x\to\infty} \sec^2(1/x)$$

$(\infty\cdot 0)$ (now $0/0$)

Answer: $\boxed{1}$

Problem 5: §4.5 #34

Work:

$$\lim_{x\to 0} (\csc x - \cot x) = \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) = \lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{\sin x}{\cos x}$$

$(\infty-\infty)$ (now $0/0$)

Answer: $\boxed{0}$

Problem 6: §4.5 # 42

Work: $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx}$ (1^∞)

let $y = \left(1+\frac{a}{x}\right)^{bx}$, then $\ln y = bx\ln(1+a/x)$

$$\text{so } \lim_{x\to\infty} \ln y = \lim_{x\to\infty} bx\ln(1+a/x) = \lim_{x\to\infty} b\,\frac{\ln(1+a/x)}{1/x}$$

$(\infty\cdot 0)$ (now $0/0$)

$$= \lim_{x\to\infty} b\,\frac{\frac{1}{1+a/x}\left(-a/x^2\right)}{-1/x^2}$$

$$= ab$$

$$\text{then } \lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx} = \lim_{x\to\infty} e^{\ln y}$$

Answer: $\boxed{e^{ab}}$