

Solutions

MAT 1320 A Assignment 2 (Due Wed. Oct. 13th, 8:30)

Student Number:

Problem 1: §3.1 #22

Answer: $\frac{dy}{dv} = ae^v - \frac{b}{v^2} - \frac{2c}{v^3}$

Problem 2: §3.1 #42

Answers: $G'(r) = \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3}$ $G''(r) = -\frac{1}{4}r^{-3/2} - \frac{2}{9}r^{-5/3}$

Problem 3: §3.2 #20

Work: $g(t) = \frac{t - \sqrt{t}}{t^{1/3}} = \frac{t - t^{1/2}}{t^{1/3}} = t^{2/3} - t^{1/6}$

So $g'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{6}t^{-5/6}$

Answer: $g'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{6}t^{-5/6}$

Problem 4: §3.2 #42

Work:

a) $h(x) = 5f(x) - 4g(x) \Rightarrow h'(x) = 5f'(x) - 4g'(x) \Rightarrow h'(2) = 5(-2) - 4(7)$

b) $h(x) = f(x)g(x) \Rightarrow h'(x) = f'(x)g(x) + f(x)g'(x) \Rightarrow h'(2) = (-2)(4) + (-3)(7)$

c) $h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \Rightarrow h'(2) = \frac{(-2)(4) - (-3)(7)}{(4)^2}$

d) $h(x) = \frac{g(x)}{1+f(x)} \Rightarrow h'(x) = \frac{g'(x)(1+f(x)) - g(x)f'(x)}{(1+f(x))^2} \Rightarrow h'(2) = \frac{(7)(1-3) - 4(-2)}{(1-3)^2}$

Answers: (a) $h'(2) = -38$ (b) $h'(2) = -29$ (c) $h'(2) = 13/16$ (d) $h'(2) = -3/2$

Problem 5: §3.3 #8

Work: $f(t) = \frac{\cot(t)}{e^t}$

$$f'(t) = \frac{(-\csc^2 t)(e^t) - e^t \cot(t)}{(e^t)^2} = -\frac{\csc^2 t + \cot t}{e^t}$$

Answer: $f'(t) = \boxed{-\frac{\csc^2 t + \cot t}{e^t}}$

Problem 6: §3.3 # 20

Work: $y = e^x \cos x$ point $(0, 1)$

$$\frac{dy}{dx} = e^x \cos x - e^x \sin x, \text{ so } y'(0) = 1 = m$$

tangent line is $y - y_0 = m(x - x_0)$ or $y - 1 = (1)(x - 0)$

Answer: the tangent line is $y = \boxed{x + 1}$

Problem 7: §3.4 # 30

Work: $f(t) = \left(\frac{t}{t^2+4}\right)^{1/2}$

$$\begin{aligned} \text{so } f'(t) &= \frac{1}{2} \left(\frac{t}{t^2+4}\right)^{-1/2} \left(\frac{d}{dt} \left(\frac{t}{t^2+4}\right)\right) = \frac{1}{2} \left(\frac{t}{t^2+4}\right)^{-1/2} \left(\frac{(1)(t^2+4) - t(2t)}{(t^2+4)^2}\right) \\ &= \frac{1}{2} \left(\frac{\sqrt{t^2+4}}{\sqrt{t}}\right) \left(\frac{4-t^2}{(t^2+4)^2}\right) \end{aligned}$$

Answer: $f'(t) = \boxed{\frac{4-t^2}{2\sqrt{t}(t^2+4)^{3/2}}}$

Problem 8: §3.4 # 32

Work: $y = \sin(\sin(\sin x))$

$$\begin{aligned} \frac{dy}{dx} &= \cos(\sin(\sin x)) \frac{d}{dx}(\sin(\sin x)) \\ &= \cos(\sin(\sin x)) \cos(\sin x) \frac{d}{dx}(\sin x) \end{aligned}$$

Answer: $\frac{dy}{dx} = \boxed{\cos(\sin(\sin x)) \cos(\sin x) \cos x}$