

Review questions

Review 1320E

Try the following exercises:

EXC 1 Suppose $f'(x) = -e^{-x^2}x$. Find the intervals where f is decreasing, and the intervals where f is increasing.

Hint: $f'(x) > 0$ if and only if $x < 0$, so f is increasing only on $(-\infty, 0)$. $f'(x) < 0$ if and only if $x > 0$, so f is decreasing only on $(0, \infty)$.

EXC 2 If $f(x) = x^3 - x$ find the intervals where f is decreasing, and the intervals where f is increasing. Moreover, find the intervals where f is concave upward, and the intervals where f is concave downward.

Hint: $f'(x) = -1 + 3x^2$ and $f''(x) = 6x$, study the sign now! Since $f''(x) = 6x > 0$ if and only if $x > 0$ we get that f is concave upward on $(0, \infty)$. Note that $f'(x) = -1 + 3x^2 > 0$ if and only if $x \in (-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$, so f is increasing on $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$.

EXC 3 Be ready to compute average rate of change, and to guess instantaneous rate of change.

You may try it for $f(x) = 2009x^2 - 1$, on intervals of the form $[2, 2.1]$, $[2, 2.01]$, $[2, 2.001]$. Can you guess the instantaneous rate of change of this function at 2?

EXC 4 Compute f' and f'' , g' where $f(x) = x^{2009}e^x$ and $g(x) = \frac{\ln(2x)}{x^{2009}e^x}$.

What rules do you apply? Do you need some properties of LOGS?

EXC 5 Solve:

$$e^{2009x^2-1} = \ln(e^7);$$

$$\ln(\ln(\ln(\ln(x - 2009)))) = 2009$$

Hint: use the inverse!

EXC 6 Show that the function $f(x) = -3 + 5x + 6x^3$ has NO tangent line with slope 4.

Hint: Note that $f'(x) = 18x^2 + 5$. Can you solve in the set of real numbers $18x^2 = -1$?

EXC 7 Find the equation of the tangent line to the function:

$$f(x) = x^4 + 6e^x + 7 \text{ at } (0, 13).$$

Hint: $4x^3 + 6e^x$ is the derivative ... plug in 0 to get the slope... for the y -intercept just plug in the equation of the tangent: $(0, 13)$...

Must know at least: derivatives of sin, cos, polynomials, exp, logs, quotient, product rules, average rate of change, instantaneous rate of change, what says the first derivative about the function (the same about the second derivative), graphs of log, exp, sin, cos. Read again the exercises done in class... Good Luck!