Lecture - 6 - OCT-17-2009

Refresh your memory: §3.6 Inverse of Trig. Fns + Their Derivatives

26/220 $Y = \cos^{-1}(\sin^{-1} t)$; $\boxed{\text{Find } y'}$

SOL: $$y' = -\frac{1}{\sqrt{1-[\sin^{-1}t]^2}} \cdot \frac{1}{\sqrt{1-t^2}}$$

— We also used Chain Rule!

$\boxed{20/220}$ $$F(\theta) = \arcsin\left(\sqrt{\sin\theta}\right) = \sin^{-1}\left(\sqrt{\sin\theta}\right)$$

SOL: $$F'(\theta) = \left(\sin^{-1}\right)'\left(\sqrt{\sin\theta}\right) \cdot \frac{1}{2}(\sin\theta)^{\frac{1}{2}-1} \cdot \cos\theta$$

$$= \frac{1}{\sqrt{1-\left(\sqrt{\sin\theta}\right)^2}} \cdot \frac{1}{2}\,\frac{1}{(\sin\theta)^{1/2}} \cdot \cos\theta$$

$$= \frac{1}{\sqrt{1-\sin\theta}} \cdot \frac{1}{2(\sin\theta)^{1/2}} \cdot \cos\theta$$

$\boxed{32/220}$ Find y' if $\tan^{-1}(xy) = 1 + x^2 y$.

SOL: $$\left(\tan^{-1}\right)'(xy) \cdot (xy)' = 0 + 2xy + x^2 \cdot y'$$

$$\frac{1}{1+(xy)^2} \cdot \left[1\cdot y + x\cdot y'\right] = 2xy + x^2\cdot y'$$

$$y + xy' = \left[1+(xy)^2\right]\cdot 2\cdot x\cdot y + \left[1+(xy)^2\right]\cdot x^2\cdot y'$$

$$y'\left[x - \{1+(xy)^2\}\cdot x^2\right] = 2xy\left[1+(xy)^2\right] - y$$

So: $y' = \dfrac{2xy\{1+(xy)^2\} - y}{x - \{1+(xy)^2\} \cdot x^2}$

MORE AT DGD! Please Attend the DGD!

§ 3.7 Derivatives of LOGarithmic Functions

THM a) $(\log_a x)' = \dfrac{1}{x \cdot \ln a}$

b) $(\ln x)' = \dfrac{1}{x}$

Pf: a) If $\log_a x = y \Rightarrow a^y = x \overset{\text{I.D.}}{\Rightarrow} a^y \cdot \ln a \cdot y' = 1 \Rightarrow$

$y' = \dfrac{1}{a^y \cdot \ln a} = \dfrac{1}{x \cdot \ln a}$.

b) $\ln e = 1$

DO: 12/226 $h(x) = \ln(x + \sqrt{x^2-1})$

SOL: Use chain Rule + §3.7 : $h'(x) =$

$= \dfrac{1}{x+\sqrt{x^2-1}} \cdot (x + \sqrt{x^2-1})' = \dfrac{1}{x+\sqrt{x^2-1}} \cdot \{1 +$

$+ \frac{1}{2}(x^2-1)^{1/2 - 1} \cdot 2x\} = \dfrac{1}{x+\sqrt{x^2-1}} \cdot \left\{1 + \dfrac{x}{\sqrt{x^2-1}}\right\}$.

DO: 14/226 $F'(y) = 1 \cdot \ln(1+e^y) + y \cdot \dfrac{1}{1+e^y} \cdot e^y$

$= \ln(1+e^y) + \dfrac{y \cdot e^y}{1+e^y}$.

4/226 $f'(x) = \left(\ln(\sin^2 x)\right)' = \dfrac{1}{\sin^2 x} \cdot 2\sin x \cos x$

$= \dfrac{2\cos x}{\sin x} = 2\cot x.$

6/226 $f'(x) = \left(\log_5 (xe^x)\right)' = \dfrac{1}{xe^x \cdot \ln 5} \cdot (xe^x)'$

$= \dfrac{1}{x\cancel{e^x} \cdot \ln 5} \cdot \left\{ e^x + xe^x \right\} = \boxed{\dfrac{1+x}{x \ln 5}}$

28/226 ! $y = \dfrac{\ln x}{x}$, POINT $(1,0)$

SOL: eq. of the tangent line: $y = mx + n$

$y' = \dfrac{\frac{1}{x} \cdot x - (\ln x) \cdot 1}{x^2}$; $m = y'(1) = \dfrac{1 - \ln 1}{1^2} = \dfrac{1}{1} = 1$

So: $y = x + n$ (eq. of tangent line)

Since $0 = 1 + n \Rightarrow n = -1$. So $\boxed{y = x - 1}$

NEW METHOD: Logarithmic Differentiation

- TAKE LN of BOTH SIDES of $y = f(x)$; Use LAWS.
- USE implicit Differentiation
- FIND y'

DO: 34/226

$\ln y = \ln\left[\sqrt{x} \cdot e^{x^2} \cdot (x^2+1)^{10}\right]$

$\ln y = \ln x^{1/2} + \ln e^{x^2} + \ln (x^2+1)^{10}$

$\ln y = \frac{1}{2} \ln x + x^2 + 10 . \ln(x^2+1)$

I.D. $\Rightarrow$ $\dfrac{y'}{y} = \dfrac{1}{2} \cdot \dfrac{1}{x} + 2x + 10 \dfrac{2x}{x^2+1} \Rightarrow y' =$

$$= \sqrt{x}\, e^{x^2} (x^2+1)^{10} \left\{ \frac{1}{2x} + 2x + \frac{20x}{x^2+1} \right\}.$$

38/226 $\quad y = x^{\cos x} \Rightarrow \ln y = \ln x^{\cos x} \Rightarrow$

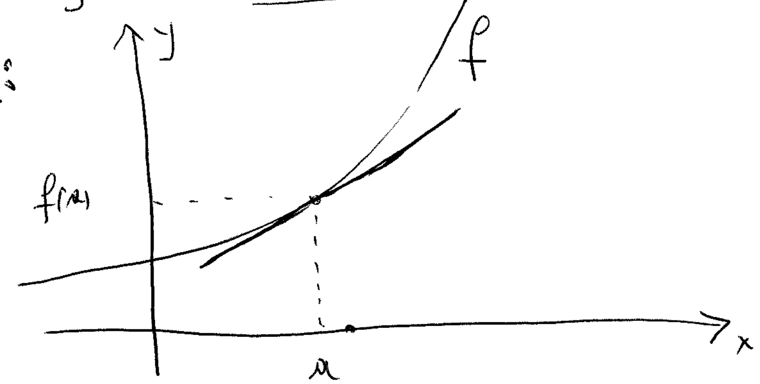
$$\ln y = \cos x \ln x \overset{\text{I.D.}}{\Rightarrow} \frac{y'}{y} = (-\sin x)\ln x + \cos x \cdot \frac{1}{x}$$

$$\Rightarrow y' = x^{\cos x} \left\{ (-\sin x)\ln x + \frac{\cos x}{x} \right\}.$$

---

§ 3.9 LINEAR Approximations and Differentials

STORY:



Def: $\boxed{f(x) \approx f(a) + f'(a)(x-a)}$ is called the LINEAR APPROXIMATION (or tangent line approximation) of f at a.

Def: $L(x) = f(a) + f'(a)(x-a)$ (treat it as a function) is called the linearization of f at a.

DO: 6,8/245

(6) $f(1) = \ln 1 = 0$; $f'(x) = \frac{1}{x} \Rightarrow f'(1) = \frac{1}{1} = 1.$

So: $L(x) = 0 + 1(x-1) = x-1.$

We WRITE: $f(x) \approx x-1$

8/245 $f(x) = x^{3/4}$; $a = 16$

Sol: $f(a) = (16^{1/4})^3 = 2^3 = 8$

$f'(x) = \frac{3}{4} x^{-1/4} \Rightarrow f'(16) = \frac{3}{4} \cdot 16^{-1/4} = \frac{3}{4} \cdot \frac{1}{\sqrt[4]{16}}$

$= \frac{3}{4 \cdot 2} = \frac{3}{8} \Rightarrow L(x) = f(a) + f'(a)(x-a) =$

$= 8 + \frac{3}{8}(x - 16)$

Q: If f is already linear, what is L??

A: . . . .

10/245 $y(x) = \sqrt[3]{1+x}$; $a = 0$; Approximate:

$\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$.

etc

CH:4 Applications of Differentiation

4.1. Read examples from 4.1, the PLAN / pg 258.

→ Read the Problem

→ DRAW . . .

→ Introduce NOTATIONS

→ Use rates . . .

→ Use equation(s)

→ Differentiate (Using Chain Rule)

→ Get the unknown RATE

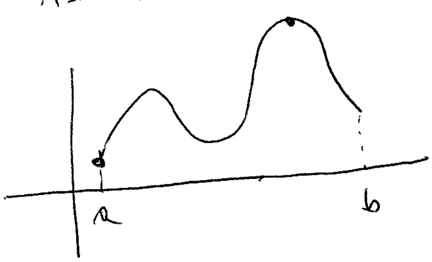
Do 6/260

DO: 14/260

§ 4.2 MAXIMUM AND MINIMUM values

Def Let c be a # in the Domain D of a function f. Then $f(c)$ is called:

- ABSOLUTE MAXIMUM value of f on D (IF) $f(c) \geq f(x)$ for all x in D
- ABSOLUTE MINIMUM value of f on D (IF) $f(c) \leq f(x)$ for all x in D.



$[a,b]=D$

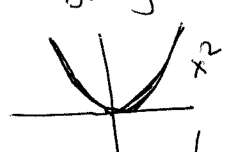
You may call them : GLOBAL MAX/MIN, OR EXTREME values of f.

Def: The number $f(c)$ is called:

- LOCAL MAXIMUM value of f (IF) $f(c) \geq f(x)$ when x is near c.
- LOCAL MINIMUM value of f (IF) $f(c) \leq f(x)$ when x is near c.

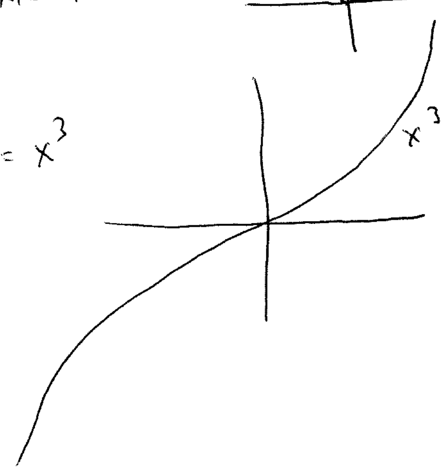
EXP: sin, cos have ∞-ly many extreme values

EXP: $f(x)=x^2$



has an ABSOLUTE (LOCAL) MIN value, BUT NO ABSOLUTE MAX value

EXP: $f(x)=x^3$



NO MAX, NO MIN

(THM) EXTREME VALUE Theorem

Any continuous function $f: [a,b] \to \mathbb{R}$

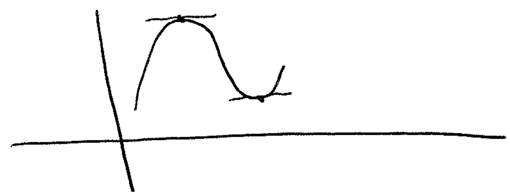
$\nwarrow$ CLOSED

attains an ABSOLUTE MAXIMUM VALUE and an ABSOLUTE MINIMUM VALUE.

Pf: Imagine

(THM) (FERMAT's Theorem) if f has a local maximum or minimum at c, and $f'(c)$ exists $\Big\}\Rightarrow$ then $f'(c)=0$

Pf. Imagine



CONVERSE NOT TRUE: Imagine $f(x)=x^3$, NO $\left\langle \begin{matrix} \text{MAX} \\ \text{MIN} \end{matrix} \right.$

$f'(x)=3x^2$; $f'(0)=0$ BUT 0 is NOT $\left\langle \begin{matrix} \text{MAX} \\ \text{MIN} \end{matrix} \right.$

(Def) A critical # of f is a # c in the domain of f such that either $f'(c)=0$ or $f'(c)$ DOES NOT exist.

THE CLOSED INTERVAL METHOD : To find the ABSOLUTE MAX and MIN values of a continuous $f: [a,b] \to \mathbb{R}$.

DO:

(1) find the values of f at the critical #s in (a,b)

(2) find $f(a)$, $f(b)$

(3) The largest of the values from step 1,2 is the ABSOLUTE MAX value ; the smallest of these values is the ABSOLUTE MIN. value.

DO: pg 268-269
