MAT 1302 E, Fall 2011

Homework 2

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For all of the questions below, you must show each step in any row reduction and state what operation you are performing at each step.

1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2k \\ 0 & -1 & k+1 \end{bmatrix}$.

   (a) **(3 points)** Find all values of k for which A is invertible.

   (b) **(2.5 points)** Find the inverse of A when $k = 0$

   (c) **(1.5 points)** Use part (b) to solve the matricial equation $Ax = b$ where

   $$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{and } b = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$$

2. **(4 points)**

   Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2011 \\ 10 & 100 \end{bmatrix}$ be 2 invertible matrices. Is there a 2×2 matrix X such that $A^{-1}B^{-1}(B + 2AX^T)BA = I_2$? If the answer is affirmative, find X.

3. **(3 points)** Let $A = \begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Compute A^{2011}.

   Hint: Find A^n for $n \leq 4$.