

MAT 1302E, Fall 2011

Homework 3

Professor: Catalin Rada

At the beginning of class 15 November 2011

For full marks show all details of your work!.

1. A certain economy is composed of 2 sectors, A and B, subject to the following conditions:

- For each unit of output, the sector A requires 0.3 units from sector A and 0.4 units from sector B.
- For each unit of output, the sector B requires 0.2 units from sector B and 0.4 units from sector A.

- (a) **(1 point)** Find the matrix C associated to this economy.
- (b) **(2 points)** Find the inverse of $(I - C)$.
- (c) **(2 points)** Find the production vector for a final demand of 50 units of sector A and 20 units of sector B.

2. Consider $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

- (a) **(1 point)** Find a basis for Col A, the columns space of A.
- (b) **(1 point)** What is the rank of A?
- (c) **(2 points)** Find a basis for Null A, the null space of A.
- (d) **(1 point)** What is the dimension of Null A?
- (e) **(1 point)** Is it true that the columns of A form a basis of \mathbb{R}^4 ? Justify your answer!

3. Determine if the following sets are SUBSPACES of \mathbb{R}^3 or not. Justify your answer!

(a) **(2 points)**

$$V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1 = x_2 x_3 \right\}$$

(b) **(2 points)**

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid 4x_1 - x_2 + 5x_3 = 0 \right\}$$

4. The following questions are independent of each other.

(a) **(2.5 points)** Assume that:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 5.$$

Compute:

$$\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ -3a_{11} & -3a_{12} & -3a_{13} \\ a_{31} + 5a_{11} & a_{32} + 5a_{12} & a_{33} + 5a_{13} \end{vmatrix}$$

(b) **(2.5 points)** Let A , B and C be matrices of size $(2, 2)$ such that

$$\det A = -2, \quad \det B = -1, \quad \det C = 1.$$

Find $\det [-3AB^4C^{-1}A^TB^{-3}(C^T)^{-1}]$.