MAT 1302E, Fall 2011

Homework 4

Professor: Catalin Rada

At the beginning of class 6 December 2011

For full marks show all details of your work!

1. Suppose that on planet Mathematics there is a Calculus alien population and an Algebra alien population. The Calculus population at week k is c_k and that Algebra population at week k is a_k. The initial population is given by $x_0 = \begin{bmatrix} c_0 \\ a_0 \end{bmatrix} = \begin{bmatrix} 3000 \\ 4000 \end{bmatrix}$. Since the population on the planet receives too many assignments, each week 10% of the Calculus population transforms (using magic) into Algebra population, while 5% of the Algebra population transforms (using magic again) into Calculus population.

(a) **(1 point)** Find the migration matrix and set up a difference equation for this situation.
(b) **(1 point)** Estimate the Calculus and Algebra population after 2 weeks.

2. **(3 points)** Solve the following equation for z.

$$z(3+2i) + (2-i) = \frac{3-i}{-i}$$

3. **(7 points)** Consider the following matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$.

(a) Find the characteristic polynomial of A.

(b) List the eigenvalues of A with their multiplicities.

c) For the eigenvalues found in part b) decribe the eigenvectors.

d) If possible, diagonalize the matrix A. In other words find an invwertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.