Faculty of Science »
Department of mathematics and statistics

MAT 1341C Test 2, 2011

17-March, 2011.

Instructor: Barry Jessup

Family Name:______________________

First Name:________________________

Student number:____________________

DGD (please circle yours):

#1– 8:30-10, MCD 146

#2– 2:30-4, MRT 250

#3– 2:30-4, CBY B012

	1	
Multiple choice answers →	2	
	3	
	4	
For the marker's use only →	5	
	6	
	[Bonus] 7	
	Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. Please record your answers in the space provided above.
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 4 points. To earn points here will be *much* more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. Let A be an 11×6 matrix such that $Ax = 0$ has only the trivial solution $x = 0$.

- What is the rank of A?
- Do the columns of A span $\mathbf{R}^{11}$?

A. 0, Yes
B. 6, Yes
C. 6, No
D. 8, Yes
E. 8, No
F. 2, Yes

2. For which value of α does the vector $(2, 3, \alpha)$ belong to the subspace of $\mathbf{R}^3$ spanned by $(1, 0, 3)$ and $(3, 2, 1)$?

A. 0
B. 6
B. -6
D. 1
E. -1
F. 1/2

3. If $B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then the second row of B^{-1} is:

A. $[1 \quad 0 \quad -1]$
B. $[-1 \quad 1 \quad 0]$
C. $[0 \quad 1 \quad -1]$
D. $[0 \quad -1 \quad 1]$
E. $[1 \quad -1 \quad 0]$
F. None of the above

4. A Norwegian Blue parrot is advised by a nutritionist to take 14 units of vitamin A, 15 units of vitamin D and 29 units of vitamin E each day. The parrot can choose from the three brands I, II and III, and the amount of each vitamin in each capsule of the various brands is given below:

	I	II	III
vitamin A	2	1	1
vitamin D	3	3	0
vitamin E	5	4	1

This parrot is not capable of taking fractions of a capsule.

a) After defining your variables, write down a system of equations in these variables, together with all constraints, that determine the possible combinations of the numbers of capsules of each brand that will provide exactly the required amounts of vitamins for the parrot.
(Do not perform any operations on your equations: this is done for you in (b). *Do not simply copy out the equations implicit in (b). You will not get any marks if you do this.*)

b) The reduced row-echelon form of the augmented matrix of the system in part (a) is:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 9 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

(Question 4 continued)

4c) Find all possible combinations of the numbers of capsules of each brand that will provide exactly the required amounts of vitamins for the parrot.

4d) If the respective costs (in cents) per capsule of brands I, II and II are 7, 3 and 2, determine the choice which will minimize the total cost each day, and give this minimum cost per day.

5. Suppose $v_1 = (1,0,2,1)$, $v_2 = (1,1,3,2)$, $v_3 = (0,1,1,1)$, $v_4 = (1,2,4,3)$ and define

$$V = \operatorname{span}\{v_1, v_2, v_3, v_4\},$$

and

$$A = [\, v_1 \quad v_2 \quad v_3 \quad v_4 \,]$$

(i.e. A is the 4×4 matrix whose jth **column** is v_j, $j = 1, \ldots, 4$).

a) Find a basis for V which is a **subset** of the given spanning set $\{v_1, v_2, v_3, v_4\}$.

b) Find a basis for $\ker A = \{x \in \mathbf{R}^4 \mid Ax = 0\}$, and hence find its dimension.

c) Find a basis for col A.

d) Extend your basis for col A from part (c) to a basis of $\mathbf{R}^4$.

6. State whether the following are true (always), or may be false, in the box after the statement. You must justify your answer: if true, explain why, if not, give an explicit example (with numbers!) to show it is false.

Suppose A is an $n \times n$ matrix such that is a vector $x \in \mathbf{R}^n$, with $x \neq 0$, and $Ax = 0$.

a) The matrix A is invertible. (You must justify your answer with an example or by using the definition and properties of the inverse: you cannot simply state a theorem or 'fact' from class.)

b) The rank of A is n. (You must justify your answer with an example or by using what you know about linear systems: you cannot simply state a theorem or 'fact' from class.)

c) The columns of A are linearly dependent. (You must justify your answer with an example or by using the definition of linear dependence: you cannot simply state a theorem or 'fact' from class.)

Hint: Write $A = [c_1 \dots c_n]$ *in block column form, write* $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, *and consider* Ax.

7. Let A and B be two matrices such that AB is defined.

a) Prove carefully that $\operatorname{rank} AB \leq \operatorname{rank} A$. (*Hint: show that* $\dim col(AB) \leq \dim \operatorname{col}(A)$.)

b) Prove carefully that $\operatorname{rank} AB \leq \operatorname{rank} B$. (*Hint: show that* $\dim \operatorname{row}(AB) \leq \dim \operatorname{row}(A)$.)