Faculty of Science »
Department of mathematics and statistics

MAT 1341A – Test 4, 2014

3-November- 2014.

Instructor: Barry Jessup. **Phone contact - extension: 3536.**

Family Name:______________________

First Name:______________________

Student number:____________________

Multiple choice answers →	1	
	2	
	3	
	subtotal	
For the marker's use only →	4	
	5	
	6	
	[Bonus] 7	
	Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 7, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
3. Questions 4 and 5 are worth 6 points each, question 6 is worth 4 points, and part marks can be earned in each. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 7 is a bonus question and is worth only 2 points. It is much more difficult to earn points in the bonus question.
4. Where it is possible to check your work, do so.
5. Good luck! Bonne chance!

1. If the coefficient matrix A in a homogeneous system of 1000 equations in 2014 unknowns is known to have rank 800, how many parameters are there in the general solution?

A. 200
B. 800
C. 1000
D. 1214
E. 2014
F. 0

2. Let A be the 15×9 coefficient matrix of a *homogeneous* linear system, and suppose that this system has infinitely many solutions with 6 parameters.

- What is the rank of A?
- Are the columns of A, considered as vectors in $\mathbf{R}^{15}$, linearly independent?

A. 0, Yes
B. 15, Yes
C. 15, No
D. 3, Yes
E. 3, No
F. 6, No

3. For a *nonhomogeneous* system of 3013 equations in 2014 unknowns, answer the following three questions:

- ∘ Can the system be inconsistent?
- ∘ Can the system have infinitely many solutions?
- ∘ Can the system have a unique solution?

A. Yes, Yes, No.
B. No, No, Yes.
C. Yes, No, Yes.
D. No, Yes, Yes.
E. Yes, Yes, Yes.
F. No, No, No.

4. Suppose $e, f \in \mathbf{R}$ and consider the linear system in x, y and z:

$$\begin{array}{ccccccc} x & - & y & + & ez & = & f \\ x & & & + & z & = & -1 \\ 3x & + & y & + & 2z & = & 1 \end{array}$$

a) If $[\,A \,|\, b\,]$ is the augmented matrix of the system above, find rank A and rank$[\,A \,|\, b\,]$ for <u>all</u> values of e and f.

(Q.4 parts (b) and (c) are on the next page...)

4b). Using part (a), find all values of e and f so that this system has

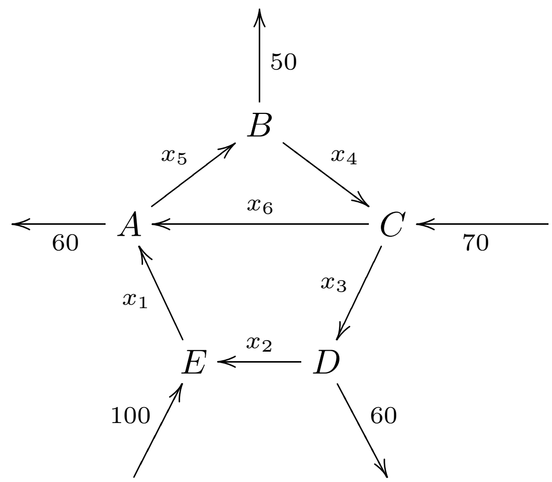
(i) a unique solution,

(ii) infinitely many solutions, or

(iii) no solutions.

4c). In case b(ii) above, give a complete geometric description of the set of solutions.

5. Consider the network of streets with intersections A, B, C, D and E below. The arrows indicate the direction of traffic flow along the **one-way streets**, and the numbers refer to the **exact** number of cars observed to enter or leave A, B, C, D and E during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



(You must justify all your answers.)

a) Write down a system of linear equations which describes the the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 6$.

(**Do not perform any operations on your equations: this is done for you in (b)**. *Do not simply copy out the equations implicit in (b). You will not get any marks if you do this.*)

(Q.7 part (b) is on the next page.)

5(b). The reduced row-echelon form of the augmented matrix of the system in part (a) is

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 1 & 60 \\ 0 & 1 & 0 & 0 & -1 & 1 & -40 \\ 0 & 0 & 1 & 0 & -1 & 1 & 20 \\ 0 & 0 & 0 & 1 & -1 & 0 & -50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

5(c). If $\overline{ED}$ were closed due to roadwork, find the minimum flow along $\overline{AC}$, **using your results from (b).**

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers!
- If you say the statement is always true, you must give a clear explanation.

a) A linear system of 5 equations in 4 unknowns is always inconsistent.

ANSWER

b) If the coefficient matrix of a homogeneous system has a column of zeros, the system has infinitely many solutions.

ANSWER

c) If the RRE form of the coefficient matrix of a homogeneous system has a row of zeros, the system has infinitely many solutions.

ANSWER

d) If A and B are 2 by 2 matrices then $AB = BA$.

ANSWER

8. [Bonus] If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix such that the vectors $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$ are linearly independent, prove carefully that rank $A = 2$. (You cannot choose the matrix A - your proof must work for every 2×2 matrix with the property above, i.e. every 2×2 matrix with independent columns.)

(You may use this page for rough work or solutions that did not fit on previous pages.)