Faculty of Science »
Department of mathematics and statistics

MAT 1341A Test 1, 2011

15-October, 2011.

Instructor: Barry Jessup.

Family Name:____________________

First Name:____________________

Student number:____________________

Multiple choice answers →	1	
	2	
	3	
	4	
For the marker's use only →	5	
	6	
	[Bonus] 7	
	Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. <u>Please record your answers in the space provided above.</u>
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 4 points. To earn points here will be more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. Let $\mathcal{P}_2 = \{\ p \mid p(x) = a + bx + cx^2,\ \text{where } a, b, c \in \mathbf{R}\}$ be the vector space of polynomial functions of degree at most 2, and consider the following subset of $\mathcal{P}_2$:

$$S = \{x^2 - 1,\ x^2 + 1,\ x - 1,\ x + 1\}$$

Which of the following statements are true?

I. S is linearly dependent
II. S is linearly independent
III. S spans $\mathcal{P}_2$
IV. S is a basis of $\mathcal{P}_2$

A. (I) and (II)
B. (I) and (III)
C. (II) and (IV)
D. (II) and (III)
E. (I), (III) and (IV)
F. (III) and (IV)

2. A vector space V has dimension 15 and W is a subspace of V in which $\{v_1, \ldots, v_8\}$ is a spanning set. Which of the following statements are always true?

I. $\dim W < 8$
II. $\dim W \leq 15$
III. $\dim W \leq 7$
IV. Any linearly independent set in W has no more than 8 vectors in it.

A. (I) and (II)
B. (I) and (III)
C. (II) and (IV)
D. (II) and (III)
E. (I), (III) and (IV)
F. (III) and (IV)

3. Which of the following are subspaces of $\mathbf{R}^3$?

(I) $\{(x,\ y,\ z) \mid x - 2y = 0\}$
(II) $\{(x,\ y,\ z) \mid xyz = 0\}$
(III) $\{(x,\ y,\ z) \mid y = 2z\}$
(IV) $\{(x,\ y,\ z) \mid x = y + 3 = z\}$

A. (I) and (II)
B. (I) and (III)
C. (II) and (IV)
D. (II) and (III)
E. (I), (III) and (IV)
F. (III) and (IV)

4. Let $\mathbf{F}([-1,1]) = \{f \mid f : [-1,1] \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $[-1,1]$. Recall that the zero of $\mathbf{F}[-1,1]$ is the function that has the value 0 for all $x \in [-1,1]$.

Define three functions in $\mathbf{F}([-1,1])$ by

$$f(x) = x, \qquad g(x) = 1 - 2x + x^2, \quad \text{and} \qquad h(x) = 1 + x^3.$$

Now let $W = \text{span}\{f, g, h\}$.

a) Show that f, g and h are linearly independent.

b) Find a basis for W and the dimension of W.

c) If $k(x) = 2 + x^2 + x^3$ show that $k \in W$.

d) What is the dimension of the subspace $Y = \text{span}\{f, g, h, k\}$?

5. Suppose $w = (0,2,1)$ and we define subspaces of $\mathbf{R}^3$ by

$$W = \{v \in \mathbf{R}^3 \mid v \times w = 0\}$$
$$U = \{v \in \mathbf{R}^3 \mid v \cdot w = 0\}$$

a) Show that $(0,2,1) \times (x,y,z) = (2z - y, x, -2x)$.

b) Use (a) to first find a spanning set for W, and then find a basis B for W.

c) Give a basis for U.

d) Extend your basis B to a basis of $\mathbf{R}^3$.

e) Give complete geometric descriptions of W and U.

6. Let u, v and w be vectors in a vector space V.

a) State carefully what "$\{u, v\}$ is linearly independent" means. (i.e., give the definition.)

Now suppose

$\star$ $\{u, v\}$ is linearly independent and that $\{u, v, w\}$ is linearly *dependent*.

Under the above assumptions, either show that the statement in (b) and (c), is always true, that is, for any V, and for any vectors u, v and w satisfying $\star$, or give a counterexample (with $V = \mathbf{R}^2$ or $V = \mathbf{R}^3$) to show that the statement in (b) and (c) isn't always true.

b) $w \in \operatorname{span}\{u, v\}$.

c) $u \in \operatorname{span}\{v, w\}$.

7. [Bonus] Suppose $\{u, v, w\}$ are three vectors in $\mathbf{R}^3$ such that $u \cdot v \times w = 2$. Prove carefully that $\{u, v, w\}$ is a basis of $\mathbf{R}^3$. *(A geometric argument involving 'volume' is not sufficient.)*