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Erratum to: Quiver varieties and Demazure modules

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Part of the proofs of Proposition 6.1 and Theorem 8.1 relied on the statement that if $w = r_{i_l} \cdots r_{i_1}$ is a reduced expression of an element of the Weyl group and $X \in B_w(\mathbf{d})$, then $X = \tilde{e}_{i_1}^{n_1} \cdots \tilde{e}_{i_l}^{n_l} X_w$ for some $n_1, \ldots, n_l \in \mathbb{Z}_{\geq 0}$. In fact, this is not true. A counterexample can be found in [1, Example 4.6.3]. However, Propositions 6.1 and 8.1 still hold. Arguments avoiding the above claim are presented below. We thank H. Nakajima and S. Naito for bringing the error to our attention.

1 Corrected proof of Proposition 6.1

We first prove that for all $X \in B_w(\mathbf{d})$, X consists of (orbits of) subrepresentations (up to isomorphism) of (x_w, t_w). Our proof is by induction on the length of w. If this length is zero, the statement is trivial. Let $w = r_{i_l} \cdots r_{i_1}$ be a reduced expression and $X \in B_w(\mathbf{d})$. If $X_{\lambda_\mathbf{d}} = \mathfrak{L}(0, \mathbf{d})$ then, by (1.2), we have

$$X = \tilde{f}_{i_l}^{n_l} \cdots \tilde{f}_{i_1}^{n_1} X_{\lambda_\mathbf{d}}$$

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for some $n_1, \dots, n_1 \in \mathbb{Z}_{\geq 0}$. Let

$$X' = \tilde{e}_{i_l}^{n_l} X = \tilde{f}_{i_{l-1}}^{n_{l-1}} \cdots \tilde{f}_{i_1}^{n_1} X_{\lambda_{\mathbf{d}}}.$$

Since $r_{i_l} w = r_{i_{l-1}} \cdots r_{i_1}$ is a reduced expression, we see that $X' \in B_{r_{i_l} w}(\mathbf{d})$ by (1.2). It follows from the inductive hypothesis that X' consists of (orbits of) subrepresentations (up to isomorphism) of $(x_{r_{i_l} w}, t_{r_{i_l} w})$. Let

$$X_w = \{[x_w, t_w]\} \in B_w(\mathbf{d}), \quad X_{r_{i_l} w} = \{[x_{r_{i_l} w}, t_{r_{i_l} w}]\} \in B_{r_{i_l} w}(\mathbf{d}).$$

Let $\mathbf{V}$, $\mathbf{V}'$, $\mathbf{V}^w$ and $\mathbf{V}^{r_{i_l} w}$ be the spaces corresponding to the representations whose orbits are points of X, X', X_w and $X_{r_{i_l} w}$ respectively. By the definition of the crystal operators in [2], we have $\mathbf{V}_i^w = \mathbf{V}_i^{r_{i_l} w}$ for $i \neq i_l$ and

$$\mathbf{V}_{i_l}^w \xrightarrow[\cong]{((x_w)_h, t_w)} \ker \left(\bigoplus_{h:\text{in}(h)=i_l} \mathbf{V}_{\text{out}(h)}^{r_{i_l} w} \oplus \mathbf{W}_{i_l} \xrightarrow{(\varepsilon(h)(x_{r_{i_l} w})_h, 0)} \mathbf{V}_{i_l}^{r_{i_l} w} \right) \stackrel{\text{def}}{=} \tilde{K}. \quad (1.1)$$

Since $X = \tilde{f}_{i_l}^{n_l} X'$, by the definition of the crystal operators given in [2] we have

$$\mathbf{V}_i = \mathbf{V}_i', \ i \neq i_l, \quad \text{and} \quad \mathbf{V}_{i_l} \cong \mathbf{V}_{i_l}' \oplus \mathbb{C}^{n_l},$$

and an open dense subset of X consists of orbits of representations (x, t) such that $\mathbf{V}'$ is x-stable and $[x', t'] \in X'$ where $x' = x|_{\mathbf{V}'}$, $t' = t|_{\mathbf{V}'}$. Now, by the stability and moment map conditions, we have that the map

$$\mathbf{V}_{i_l} \stackrel{(x_h, t)}{\hookrightarrow} \ker \left(\bigoplus_{h:\text{in}(h)=i_l} \mathbf{V}_{\text{out}(h)}' \oplus \mathbf{W}_{i_l} \xrightarrow{(\varepsilon(h) x_h', 0)} \mathbf{V}_{i_l}' \right) \stackrel{\text{def}}{=} K \quad (1.2)$$

is injective. Since (x', t') is a subrepresentation (up to isomorphism) of $(x_{r_{i_l} w}, t_{r_{i_l} w})$, we have (after replacing representations by different orbit representatives if necessary) $K \subseteq \tilde{K}$. Thus, for (x, t) in an open dense subset of X, (x, t) is isomorphic to a subrepresentation of (x_w, t_w) by (1.1), (1.2) and the inductive hypothesis. As in the original proof, we see that the set of orbits of subrepresentations of (x_w, t_w) is closed and thus X consists entirely of such representations.

The proof of the converse statement, that if every point of X is an orbit of a subrepresentation of (x_w, t_w) then $X \in B_w(\mathbf{d})$, is as in the original.

2 Corrected proof of Theorem 8.1

We prove the case of w_n^-. The case of w_n^+ is analogous. By (1.2),

$$P_{w_n^-}(\lambda) = \{\tilde{f}_{i_n}^{k_n} \cdots \tilde{f}_{i_1}^{k_1} P_\lambda \mid k_1, \dots, k_n \in \mathbb{Z}_{\geq 0}, \ i_j \equiv j \mod 2\}.$$

Suppose $P \in P_{w_n^-}(\lambda)$. Since the blocks of any stack in a Young pyramid must alternate color, we see that the maximum height of any stack in P is n and the maximum height of any stack with bottom block of color 0 (e.g. the stacks in the column one) is $n-1$. Since P is 1-reduced, the heights of stacks must strictly decrease as we move east. Therefore, the height of any stack in the ith column is less than or equal to $n-i$. Therefore P is a subpyramid of $P_\lambda^{w_n^-}$.

The proof of the converse statement, that any subpyramid of $P_\lambda^{w_n^-}$ is contained in $P_{w_n^-}(\lambda)$, is as in the original.

References

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