Ottawa Mathematics Conference
May 1-2, 2008

ABSTRACTS

Invited Speakers

MONICA NEVINS
University of Ottawa

Representation Theory and Applications

The representation theory of groups — from finite groups to Lie groups to algebraic groups over interesting fields — is a beautiful and elegant subject which has found applications to a variety of areas, in and out of mathematics. In this hour, we would like to share some of the beauty of representation theory, as well as some of its more tantalizing applications.

BRETT STEVENS
Carleton University

Locating and avoiding errors in software testing

The past 15 years have seen a great deal of research on the combinatorial object used in reliability testing: the covering array. In the past few years the model has been adapted to incorporate application relevant issues: known non-interactions, mixed alphabet sizes and mixed strength. We will discuss two very recent adaptations which are surprisingly related: avoiding pairs of forbidden interactions and locating the exact location of an error when its existence is discovered. We will discuss the complexity of these questions, combinatorial characterizations of feasibility and arrays and algorithms that solve these problems in some instances.

Abstracts of Contributed Talks

MO'TASSEM AL-ARYDAH
University of Ottawa

Existence of positive solution to a nonlinear system of PDE in a domain with a triple-phase boundary

We consider a system of nonlinear partial differential equations describing the reaction-diffusion dynamics near a triple-phase boundary in catalyst layer of hydrogen fuel cells. The system couples the dynamics in free air, porous and surface domains. Using certain a priori estimates, variational methods and fixed point theorem we prove the existence of a positive weak solution.

JAMAL HUSSAIN ALSMAIL
University of Ottawa

Two Dimensional Hydrogen Fuel Cell: Channel Optimal Shape Design

We consider a two dimensional model of dry hydrogen fuel cell, the cathode side. This part of the cell involves two layers: the air channel and the graphite diffusive layer. Our objective is to find the optimal shape of the air channel such that the following constraints are met:

1. The oxygen distribution is uniform on the cathode membrane,
2. The total oxygen is maximized on the cathode membrane.

We achieve this goal by implementing shape calculus and the adjoint method.

KARIN ARIKUSHI
Carleton University

Almost perfect nonlinear polynomials

Cryptographic systems such as AES and DES, which utilize S-boxes, must be resistant to differential cryptanalysis in order to be effective. Almost Perfect Nonlinear (APN) functions are, by definition, optimally resistant to differential attacks and are good candidates for use in S-boxes. However, it is still an open problem to find and classify all APN functions over finite fields.

Until recently, the only known APN polynomials were power functions, i.e. functions of the form $f(x) = x^d$. In the last five years, new binomial and multinomial examples have also been found. We will consider a recent class of APN binomials over $\mathbb{F}_{3^n}$ found by Ness and Helleseth. They proved the APN property using quadratic characters, so we will present a modified approach using different characters. This is still work in progress, but we hope that this method will yield a new class of APN multinomials.

ROBERT BAILEY
Carleton University

Distance enumerators of permutation groups

The *distance enumerator* of an error-correcting code is a polynomial which "counts" the number of codewords from a fixed word w. (For linear codes, this is often called the *weight enumerator*.) We consider the situation where the code is a group of permutations, in which case the distance enumerator is related to a more well-known polynomial, the cycle index. This is joint work with J.P. Dixon (London/Sydney).

SAMINA BASHIR
University of Ottawa

Automorphisms of simple anti-Jordan pairs

We will describe anti-Jordan pairs, their classification and then their automorphism groups.

AMY CAMERON
University of Ottawa

The Multi-two-edge Connected Subgraph Problem

The minimum cost two-edge connected subgraph problem is an important problem in network design, and has many practical, cost-saving applications, such as the design of reliable communication and transportation networks. This talk will present a 3/2-approximation algorithm for the multi-two-edge connected subgraph problem. Previously, 2 was the best known approximation guarantee.

EMILIE DUFRESNE
Queen's University

Separating invariants and finite reflection groups

The idea of separating invariants comes from the desire to distinguish the orbits of a group action on a finite dimensional vector space. Roughly speaking, a separating algebra is a subalgebra which distinguishes the orbits of a group action equally well as the whole ring of invariants. Separating algebras are often better behaved than the ring of invariants. In this talk we give necessary conditions for the existence of polynomial separating algebras (the "best" possible behavior).

LAURA DUMITRESCU
University of Ottawa

The asymptotically optimal estimating equation for longitudinal data

In this talk, we introduce a conditional marginal model for longitudinal data, in which the residuals form a martingale difference sequence. This model allows us to consider a rich class of estimating equations, which contains several generalized estimating equations proposed in the literature. Using the approach of Heyde (1997), we identify a sequence of "asymptotically optimal" equations within this class. Next, we prove the existence of a sequence of strongly consistent estimators for the regression parameter, defined as roots of the sequence of asymptotically optimal equations. This talk is based on joint work with Raluca Balan (University of Ottawa) and Ioana Schiopu-Kratina (Statistics Canada).

JANE HE
Carleton University

The path ideal of a tree and its properties

Given a tree G, we consider the path ideal $I_t(\Gamma)$, that is, the ideal where every generator corresponds to a path of length t in Γ. When this path ideal is regarded as a facet ideal of a simplicial complex, that is, we view every generator of the path ideal as a facet of this simplicial complex, we show this simplicial complex is actually a simplicial tree. By using a property of a simplicial tree due to Faridi, we prove that $R/I_t(\Gamma)$ is sequentially Cohen-Macaulay.

RATNADHA KOLHATKAR
University of Ottawa

Some properties of affine surfaces with trivial ML-invariant

ML-invariant, named after Leonid Makar-Limanov, provides a powerful tool in the study of rings or affine varieties. We will state some properties of affine surfaces with trivial ML-invariant, and then sketch a proof of the fact that, the kernel of every nonzero locally nilpotent derivation of such a surface is a polynomial curve over the base field.

FOK-SHUEN LEUNG
University of Waterloo

Rational points on split del Pezzo surfaces

Describing integer solutions to Diophantine equations, or rational points on algebraic varieties, is one of math's oldest and most fundamental themes. In 1989, Yuri Manin made a conjecture predicting the density of rational points on del Pezzo surfaces-rational varieties isomorphic to either $\mathbb{P}^1 \times \mathbb{P}^1$, or $\mathbb{P}^2$ blown up along up to nine points. Proofs of the conjecture exist for surfaces of relatively high degree, and for singular surfaces of low degree. Smooth surfaces of low degree have proven less tractable.

I will survey the state of Manin's conjecture, and describe some recent work giving a partial proof of the conjecture for a family of split non-singular quartic del Pezzo surfaces. No special knowledge of algebraic geometry will be required.

JEFFREY MORTON
University of Western Ontario

2-Vector Spaces and Groupoids

A Kapranov-Voevodsky 2-vector space is a setting for a categorical analog of linear algebra. In this talk, I will describe these, and demonstrate a construction for getting such a 2-vector space from any essentially finite groupoid X. A related process gives "2-linear maps" between such 2-vector spaces from spans of groupoids. I will also suggest a motivation for this in terms of "categorified quantization".

LIDIA NIKITINA
Carleton University

A mathematical description of internal gravity waves forced by topography

Waves in fluids can be simulated with mathematical models based on Newton's laws of motion, with the waves being represented as sinusoidal functions superimposed on a steady basic flow. In geophysical fluid dynamics, numerical and analytical solutions of such mathematical models help us to understand the behaviour of waves in the atmosphere and oceans. In my research I am using analytical methods, such as multiple scaling and asymptotic expansions, to study the nonlinear dynamics of gravity waves forced by topography, such as an isolated mountain or a mountain range. In my talk I will explain how the solutions are derived and talk about their physical interpretation. (This is joint work with L.J.Campbell.)

MEHMETCIK PAMUK
McMaster University

Homotopy self-equivalences of 4-manifolds with free fundamental group

I will first talk about the braid constructed by Hambleton and Kreck to study the group $\mathrm{Aut}_\bullet(M)$ of homotopy classes of base-point preserving homotopy self-equivalences of a 4-manifold M and then give an explicit description of $\mathrm{Aut}_\bullet(M)$ when the fundamental group of the manifold is a free group.

EMILY REDELMEIER
Queen's University

Real Wishart matrices and nonorientable surfaces

The combinatorics of orientable surfaces can be used to calculate some of the statistics of complex Wishart matrices, such as moments and fluctuations around their asymptotic limits. I will look at adapting these tools to the real case, in which nonorientable surfaces as well as orientable ones appear. In particular, I will look at adapting the Kreweras complement (information about faces determined by information about vertices and edges), and the algebraic methods of calculating it.

PATRICK REYNOLDS
Queen's University

Hamiltonian systems of hydrodynamic type

What does it mean for a system of PDEs to be Hamiltonian? I'll briefly review this idea before moving on to consider Hamiltonian systems of PDEs that are "of hydrodynamic type", in particular the construction of an interesting class of Poisson brackets for such systems. There are surprising connections here with Riemannian geometry, and I'll try to highlight these.

NOUSHIN SABETGHADAM
Concordia University

The cusp widths of the congruence subgroups

It is a remarkable fact that the set of the cusp widths of any congruence subgroup is closed under taking greatest common divisors and least common multiples. This result was proved first by Larcher by introducing a special family of congruence subgroups called Larcher subgroups. Cummins and I computed the signature of a larger class of congruence subgroups which contains Larcher subgroup. As an application this computation allowed us to reprove the Larcher result quoted above.

CHARLES STARLING
University of Ottawa

Aperiodic Tilings

The study of aperiodic tilings of the plane spans many branches of mathematics including operator theory, dynamics and algebraic topology. I will discuss the use of topological invariants in the study of tilings and how rotational symmetries present in a tiling may help us to better understand these invariants.

COLIN WEIR
Carleton University

Hyperelliptic function fields

Hyperellitic curves are objects of algebraic geometry and as such, typically described over algebraically closed fields. One invariant between equivalent hyperelliptic curves is their field of rational functions. These are commonly referred to as hyperelliptic function fields. In addition, the points on the curve are in one-to-one correspondence with the maximal subrings of their rational function field. If we disregard the geometry and relax the condition of algebraic closure, the problem then becomes finding and describing all the maximal subrings of a hyperelliptic function field.

We will briefly elaborate on the motivation for describing these maximal subrings. We will then show how each maximal subring gives an absolute value on the function field. With these absolute values we will then construct completions and use them to find and describe all maximal subrings.

I will attempt to keep this talk as accessible as possible by giving examples and descriptions of all terminology along the way.

ANGELIKA WELTE
University of Ottawa

Non-commutative coordinates for universal central extensions

Lie algebras are non-commutative and non-associative algebraic structures which play an important role in geometry, algebra and mathematical physics. In the classical theory outstandingly nice results have been proven for the special linear Lie algebras $sl_n(\mathbb{C})$ and their representations. Most importantly for our work there are: Homological algebra and Whitehead's Lemmas, presentations and Serre's Theorem, combinatorics and root systems. We will explain how those results can be reformulated for special linear Lie algebras with coordinates in a non-commutative or non-associative algebra $\mathcal{A}$. Our approach reveals again an interesting connection between properties of the coordinate algebra, homological data of the Lie algebra and also combinatorial properties of the root system. It turns out that not only the Lie algebra, but also its universal central extension can be coordinatized by $\mathcal{A}$.

ENXIN WU
University of Western Ontario

Deformation of A-infinity algebras

The idea of deformation theory (deformation quantization) mainly came from geometry and physics. In the 1960's, Gerstenhaber used this idea in pure algebra to study the deformation theory of various algebras. For an associative algebra, Gerstenhaber connected the deformation theory with the second and third Hochschild cohomology groups. We call this the classical deformation theory of an associative algebra, and try to extend it to the context of A-infinity algebras, with the aim of deforming an associative algebra to an A-infinity algebra so that we may use the techniques of A-infinity algebras to solve problems about associative algebras. This work is in progress. In the talk, because of the complexity of A-infinity algebras, we will mainly focus on the known results about the deformation theory of associative algebras, and briefly introduce the basic definitions, properties and corresponding deformation theory of A-infinity algebras.